**Problem statement:** Solve1D unsteady diffusion equation numerically and analytically

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

With periodic boundary condition in the domain [0, 1] and initial condition

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

**Analytical solution using separation of variables:**

Analytical solution is calculated using separation of variables technique. Here the dependent variable is expressed as a combination of a function of each independent variable, resulting in a system of ODE which can be solve d analytically.

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Therefore differentiating (3) twice w.r.t x, we have

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

And w.r.t t,

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Substituting (4) and (5) in (1)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (6) |

Equating (6) equation to -λ2

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

We get the following ODEs

|  |  |  |
| --- | --- | --- |
|  |  |  |

General solution for first equation with imaginary roots and zero real part is given by

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

For the second ODE the solution is,

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

Therefore, the solution for u is

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

Using following boundary condition and initial condition

, ,

Substituting we get and writing

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

Substituting we get

i.e., and

The solution would be the linear combination of all values of for different values of . The unique solution is obtained by equating to the initial condition.

Therefore

The final solution is

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

**Comparison of analytical and numerical results:**

